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Noise induced on an electric antenna and its effective length in solar wind: Application to CLUSTER observations

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Abstract

Two problems related to the performance of an electric antenna in the solar wind are addressed. Namely, the noise induced on the antenna and its effective length at the close vicinity of the plasma resonance. The antenna is a double sphere dipole on the Cluster spacecraft and data are obtained from the WBD instrument. The noise data are interpreted to estimate the plasma parameters. In case the antenna is receiving regular signals, including quasi-harmonic signals coming from a distant source either of natural or artificial origin, it is its effective length that determines the voltage on antenna terminals. We show that the effective length grows by more than one order of magnitude for waves at the local plasma frequency propagating downstream to the receiver.

Keywords: Quasi-thermal noise spectroscopy; Effective length of a dipole; Resonance conditions

1. Introduction

This paper treats two problems related to investigation of plasma waves in CLUSTER wave experiments in the streaming solar wind.

(i) The analysis of a *noise induced on a dipole antenna* in a non-equilibrium plasma of the solar wind, quasi-thermal noise (QTN) spectroscopy. The QTN spectroscopy has been successfully used in measurements in the solar wind by Meyer-Vernet and collaborators, (see, e.g. Issautier et al., 1999; Meyer-Vernet et al., 2000; Meyer-Vernet and Perche, 1989). Our aim here is to look at the high frequency resolution noise spectra very close to the local plasma frequency in the model of two temperature plasma and compare our analytical results to the measurements of WBD (wideband) instrument on the Cluster spacecraft.

(ii) The study of *effective length of the double sphere dipole antenna* under resonance conditions is related to the reception of quasi-regular signals (quasi-harmonic waves) coming to the receiving antenna from a distant source. It allows for meaningful conversion of the voltage measured on the antenna terminals to the electric field of incident waves. The general treatment was given by Chugunov (2001) and was successfully applied to two point measurements (transmitter ↔ receiver, James, 2000) of waves around the lower oblique resonance in the ionosphere (Chugunov et al., 2003).

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2. Theoretical background

Radiation power spectral density on a receiving antenna is expressed as an integral over wave vector space

$$\mathscr{E}_\omega^2 = \frac{2}{\pi}\int \frac{(2\pi)^3 W_k^{\mathrm{inc}}(\omega,\vec{k})}{k^2}\,\mathrm{d}R_k, \tag{1}$$

This expression is correct for both the noise and regular signals on antenna terminals (Andronov and Chugunov, 1975; Chugunov, 2001). Here,

$$\begin{aligned}\mathrm{d}R_k &= \frac{(2\pi)^5}{\omega-\vec{k}\cdot\vec{u}}|\vec{k}\cdot\vec{j}_0(\vec{k})|^2\delta(\varepsilon_\mathrm{l}(\omega-\vec{k}\cdot\vec{u},\vec{k}))\,\mathrm{d}k\,\mathrm{d}\Omega\\ &= \frac{\omega}{\omega-\vec{k}\cdot\vec{u}}\mathrm{d}R_k^*\end{aligned} \tag{2}$$

and $\mathrm{d}R_k^*$ is a differential resistance of radiation in the resonant mode, and the longitudinal permittivity

$$\varepsilon_\mathrm{l}(\omega-\vec{k}\cdot\vec{u},\vec{k}) = (k_i\varepsilon_{ij}k_j)/k^2 = 0 \tag{3}$$

gives the dispersion equation of electrostatic waves. We use spherical coordinates k, θ, ϕ, so that $k_z = \cos\theta, k_\perp = \sin\theta$ and the dispersion is given by

$$(\omega - k_z u)^2 - k_z^2 v_\mathrm{T}^2 - k_\perp^2 v_\mathrm{T}^2 - \omega_\mathrm{p}^2 = 0, \tag{4}$$

$\vec{j}_0(\vec{k})$ is the Fourier component of the current density distribution of unit amplitude on the antenna, $\mathrm{d}\Omega = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi$ is the solid angle, u the solar wind stream velocity, θ is the angle between vectors $\vec{u}$ and $\vec{k}$, φ is the azimuthal angle, ω_p the electron plasma frequency, v_T the electron thermal velocity and $\omega = 2\pi f$, f is the frequency of radiation. As already stated the formula (1) is valid for any incident radiation of energy density $W_k^{\mathrm{inc}}(\omega,\vec{k})$ in non-equilibrium uniform media under the condition that the antenna plasma system is in a stationary state. In case of noise voltage, the spectral energy density in (1) is expressed by means of an effective temperature $W_k^{\mathrm{inc}}(\omega,\vec{k}) = k^2T_{\mathrm{eff}}/(2\pi)^3$ and

$$T_{\mathrm{eff}}(\omega,\vec{k}) = \frac{T_\mathrm{c}\Im\varepsilon_{\mathrm{lc}} + T_\mathrm{h}\Im\varepsilon_{\mathrm{lh}}}{\Im\varepsilon_{\mathrm{lc}} + \Im\varepsilon_{\mathrm{lh}}} \approx \frac{T_\mathrm{h}\Im\varepsilon_{\mathrm{lh}}}{\Im\varepsilon_{\mathrm{lc}} + \Im\varepsilon_{\mathrm{lh}}} \approx T_\mathrm{h}. \tag{5}$$

Here, we assume that the plasma is composed of two populations, a "cold" core component with the temperature T_c, density N_c, dielectric permittivity $\varepsilon_{\mathrm{lc}}(\omega-\vec{k}\cdot\vec{u},\vec{k})$, and a hot halo component with $T_\mathrm{h} \gg T_\mathrm{c}$, $N_\mathrm{c} \gg N_\mathrm{h}$ and $\varepsilon_{\mathrm{lh}}(\omega-\vec{k}\cdot\vec{u},\vec{k})$. $\Im$ denotes the imaginary part. Under these conditions the wave dispersion is given by the "cold" component alone. Such a model was used already in the above cited works (Issautier et al., 1999; Meyer-Vernet et al., 2000; Meyer-Vernet and Perche, 1989). Here, we restrict our investigation to the close vicinity of the plasma resonance so that only Langmuir waves described by Eq. (4) come into play. This approach both simplifies the calculations and retains essential features of dispersive properties of the solar wind plasma.

The antenna on Cluster is a double sphere dipole with the spheres at a distance $2L$ = 88 m apart and makes an angle α with the stream velocity. In (1), we first integrate over the module of the wave vector with the aid of delta function, which selects, according to the dispersion relation, the wave modes incident on the antenna in the direction given by angles θ, ϕ. Then the charge density distribution corresponding to two charges of unit amplitude separated by a distance $2L$ is integrated over the azimuthal angle ϕ. We arrive at the expression for the spectral density of the noise voltage used in further comparison with the measured data.

$$\begin{aligned}V_\omega^2 &= \frac{2}{\pi}\frac{T_\mathrm{h}}{u}\int_{x_*}^{1}\frac{\mathrm{d}x}{\sqrt{x^2+\varepsilon_0/M^2}}\\ &\quad\times\sum\nolimits_{j=1,2}\left[1-\cos(L_z\kappa_j x)J_0(L_\perp\kappa_j\sqrt{1-x^2})\right].\end{aligned} \tag{6}$$

Here, $x_* = 0$ if $\omega > \omega_\mathrm{p}$ and $x_* = \sqrt{|\varepsilon_0|/M^2}$ if $\omega < \omega_\mathrm{p}$, $\varepsilon_0 = (1-\omega_p^2/\omega^2)$, $\kappa_{1,2} = -x \pm \sqrt{x^2+\varepsilon_0/M^2}$ are the (normalized) roots of the dispersion relation (4), $L_z = 2M\omega L\cos\alpha/v_\mathrm{T}$, $L_\perp = 2M\omega L\sin\alpha/v_\mathrm{T}$, $M = u/v_\mathrm{T}$, J_0 denotes the Bessel function. Eq. (6) can be rewritten as $V_\omega^2 = \frac{2}{\pi}T_\mathrm{h}R_{\mathrm{rad}}(\omega,\alpha)$, where $R_{\mathrm{rad}}(\omega,\alpha)$ is the antenna radiation resistance due to plasma waves in a drifting plasma. In this way (6) is an analogue of the Nyquist relation in our problem. In principle two other facts come into play: the shot noise due to both "cold" and "hot" particle populations and the finite input impedance of the receiver. As our estimations showed the latter is nearly by two orders of magnitude higher than the resistance due to radiation and shot effect. Also the shot noise contribution is several times lower than that of plasma waves. Consequently, these two effects are not taken into account in the interpretation of the measured spectra as they stay in the margin of their accuracy (see the next paragraph).

In case of a quasi-harmonic wave coming from a distant source to the receiver the open circuit voltage is $V^2 = \mathscr{E}_\omega^2\Delta\omega$ ($\Delta\omega$ is the receiver bandwidth) and $W_k^{\mathrm{inc}}(\omega,\vec{k})$ is expressed as

$$W_k^{\mathrm{inc}}(\omega,\vec{k})\Delta\omega = \frac{\Pi_{\mathrm{gr}}^{\mathrm{inc}}}{\Delta\Omega_{\mathrm{gr}}\cos\vec{k}{}^\wedge\vec{v}_{\mathrm{gr}}}, \tag{7}$$

which is used in (1) for our calculations of the antenna effective length. The Poynting vector $\Pi_{\mathrm{gr}}^{\mathrm{inc}}$ is directed along the radius vector from the emitter point to the receiver point, $\vec{k}{}^\wedge\vec{v}_{\mathrm{gr}}$ is the angle between the vectors $\vec{k}$ and $\vec{v}_{\mathrm{gr}}$, the group velocity, and $\Delta\Omega_{\mathrm{gr}}$ is the solid angle in the direction of group velocity. Once again the integration over the module of the wave vector is performed with the aid of delta function, in the remaining integral over the solid angle in the wave vector space we change to solid angle in the coordinate space and the integration goes only over a (small) solid angle around the direction of the incoming signal.

It may be shown that the amplitude of rms open circuit voltage, $U = \sqrt{2}V$, is given as a product of the amplitude of the incident wave at the point of reception and the effective length of the receiving antenna

$$U = EL_{\mathrm{eff}}(\omega, \gamma_0). \tag{8}$$

In this paper, we present an expression for L_{eff} for frequencies above the plasma frequency

$$L_{\mathrm{eff}} = \frac{\omega_{\mathrm{pe}}}{\omega} \frac{|\cos\gamma_0 \cos\alpha - \overline{M}\cos\alpha + \sin\gamma_0 \sin\alpha \cos\phi_0|}{(1 - \overline{M}\cos\gamma_0)^{1/2}(1 - 2\overline{M}\cos\gamma_0 + \overline{M}^2)^{3/4}} \times \frac{|\sin k_* L_*|}{k_* L_*} 2L, \tag{9}$$

where $k_* L_* = \frac{\omega L}{v_{\mathrm{T}}}\sqrt{M^2 + \varepsilon_0}|\cos\gamma_0 \cos\alpha - \overline{M}\cos\alpha + \sin\gamma_0 \sin\alpha \cos\phi_0|$ results from the projection of the wave vector k_* of the incident plasma wave on the oriented antenna length, $M/\overline{M} = \sqrt{M^2 + \varepsilon_0}$. In a spherical system of coordinates (with z-axis along the wind stream velocity) the antenna direction makes the angle α with z-axis, ϕ_0 gives its azimuthal position and γ_0 is the angle of arrival of the signal to the antenna with respect to the stream velocity (the angle of the group velocity of the incident wave). The wave number k_* is that of the incident wave.

Essential details and results are presented in the following section.

3. Results and discussion

Figs. 1(a) and (b) present noise spectra received near the plasma frequency in a streaming solar wind by the WBD instrument on CLUSTER and the best fit of computed spectra according to the expression (6). As our analytical calculations show the plasma resonance is in no way apparent in these spectra. The maximum lies between the plasma frequency and the cutoff at $\sim f_{\mathrm{p}} - f_{\mathrm{p}}M^2/2$, and its position depends on the antenna orientation with respect to solar wind velocity. The amplitude in the frequency band concerned and under conditions mentioned above is proportional to the ratio of the temperature of the hot plasma component T_{h} to the solar wind velocity u. In the fitting procedure we chose the solar wind velocity u as an external parameter. It's value of 280 km/s was obtained by the CIS instrument of the same spacecraft. The discrepancy between the model prediction and observed spectra for frequencies greater than f_{p} does not allow to perform a full 3 parametric fit in T_{c}, u and f_{p}. Therefore, we have to fix the value of one of the parameters to an a priori known value to reduce the number of degrees of freedom. We chose the solar wind velocity u because its measurements are relatively exact and robust comparing to the PEACE measurements of electron temperature. This choice also resulted in the best agreement between the fitted curve and observed spectrum.

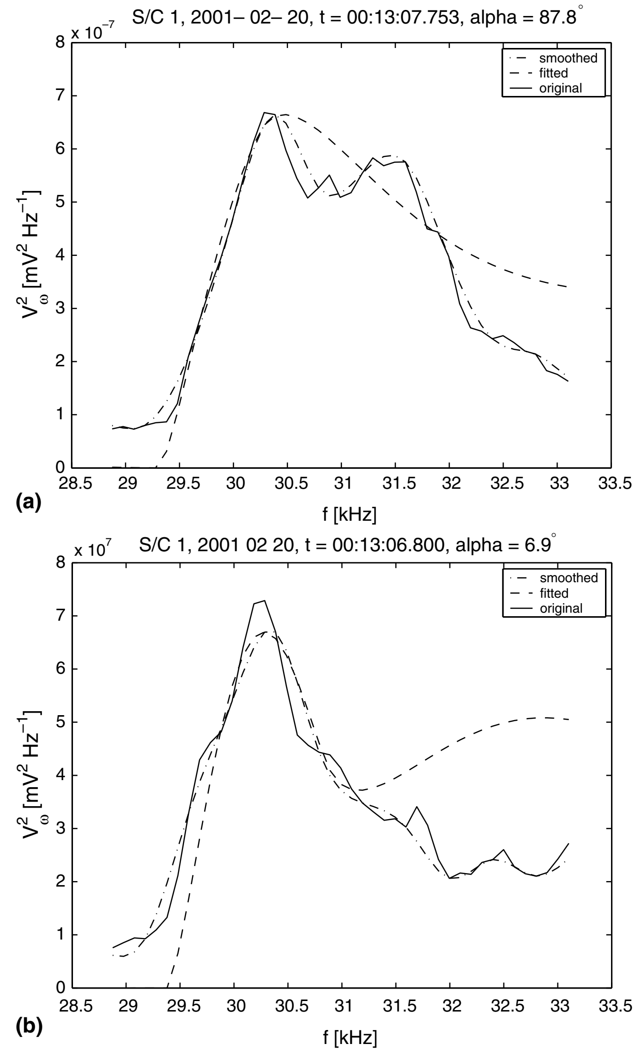

Fig. 1. (a) Fitting the computed and measured spectra, antenna perpendicular to the solar wind. (b) Fitting the computed and measured spectra, antenna aligned with the wind.

In Fig. 1(a), the antenna is perpendicular to the wind velocity (angle 87.8 ± 7.5°), in Fig. 1(b) the antenna is aligned to the stream (angle 6.9 ± 7.5°). The best fit in Fig. 1(a) gives cutoff at 29.2 kHz, plasma frequency at 30.3 kHz, $M = 0.32$, $T_{\mathrm{c}} = 7.5 \times 10^4$ K, $T_{\mathrm{h}} = 1.3 \times 10^4$ K, in Fig. 1(b), these numbers are, respectively, 29.3, 30.9 kHz, 0.27, 4.9 and 1.1×10^4 K. The spectra were obtained from the WBD instrument. The instrument provides 2180 samples in 10 ms intervals which are 70 ms apart. Three consecutive spectra were averaged, which gives about 15° in angular resolution and additional smoothing was applied to keep statistical error at about 20% and frequency resolution is at about 400 Hz. The WBD receiver noise is below 5 μV (Gurnett et al., 1997), our spectra are above ∼30 μV.

The spectra fit well from the cutoff, identified already in (Issautier et al., 1999), to the plasma frequency. The discrepancy at higher frequencies remains unexplained in this limited one case study. Our estimation of Landau damping of the plasma modes in this frequency range seem to show that this can be neglected, so an attention to this problem will be paid in a follow-up with a larger set of data. Also, the calculated spectrum on the antenna perpendicular to stream velocity shows a single maximum instead of two in measured spectrum. Here, it should be borne in mind that the discrepancy is in the range of 20% accuracy of the spectral amplitude, which is the result of compromising between the angular and frequency resolution. In the framework of our model of the streaming otherwise isotropic plasma we cannot speculate on effects of possible modes in a magnetized plasma though the angle between the solar wind velocity and the magnetic field is only several degrees so that the antenna in this case is nearly perpendicular to it. Further refinement of both the procedure for data interpretation and the plasma model are being developed.

The expression (9) determines the effective length of a double sphere dipole antenna with the spheres $2L$ apart, for frequencies above the plasma frequency. It is seen that $L_{\rm eff}$ is a product of the antenna geometric length and of the term that represents an excitation coefficient for the radiation field of a longitudinal wave in a given direction, which is the dipole's (non-normalized) radiation pattern.

In a non-streaming plasma, $M = 0$, and for a short dipole, $k_*L_* \ll 1$ (dipole's length much shorter than the wavelength of the incident field), we obtain from (9) $L_{\rm eff} = \frac{\omega_{\rm p}}{\omega}|\cos\gamma_0\cos\alpha + \sin\gamma_0\sin\alpha\cos\phi_0|2L$, i.e., the effective length is proportional to the geometrical length and to the factor that characterizes the polarization of the medium and vector relations of the fields. In fact in this case the antenna measures the potential difference between two points in a uniform field of the incident wave. In general, even for a short dipole, the effective length differs from the geometrical one. It shows itself for $\omega \to \omega_{\rm p}$ and angles of arrival $\gamma_0 \to 0$, that is for waves propagating downstream. In such a case, $\varepsilon_0 \ll M^2$, $L_{\rm eff}(\omega, \gamma_0 = 0) = 2M^2\cos\alpha/\varepsilon_0 2L$ and the effective length grows sharply near the plasma frequency.

To help the understanding of changes in the effective length $L_{\rm eff}$ that follows, the next Figs. 2(a)–(c) show wave number surfaces (normalized by the Debye length) for Langmuir waves in a streaming plasma. These surfaces are very useful for insight into essential features involved in radiation or reception of waves. The surfaces are elliptic with slightly longer axis along the stream velocity u (z-axis), for the conditions prevailing in the solar wind ($M = u/v_{\rm T} \sim 1/3$). They exist also below the local plasma frequency. The surface shrinks to a point, at a cutoff at $\omega_{\rm p}(1 - M^2)^{1/2}$. Fig. 2(a) shows that for $\omega > \omega_{\rm p}$ the surface is close to that in an isotropic medium, the group velocity makes an acute angle with phase velocity. Here the $L_{\rm eff}$ is of the order of antenna geometric length. When $\omega < \omega_{\rm p}$ (Fig. 2(c)), there exist a point, where the group and phase velocity are perpendicular. There two waves coalesce: the one with growing acute angle between the phase and group velocity, the other with decreasing obtuse angle between the velocities. The interference of these two waves makes the $L_{\rm eff}$ to grow. Finally, the most remarkable growth of $L_{\rm eff}$ occurs at $\omega \to \omega_{\rm p}$, for incident radiation coming downstream to the receiver. As seen from Fig. 2(b) this is due to a coherent interference of an infinite set of planar waves that contribute to the energy flow along the stream velocity.

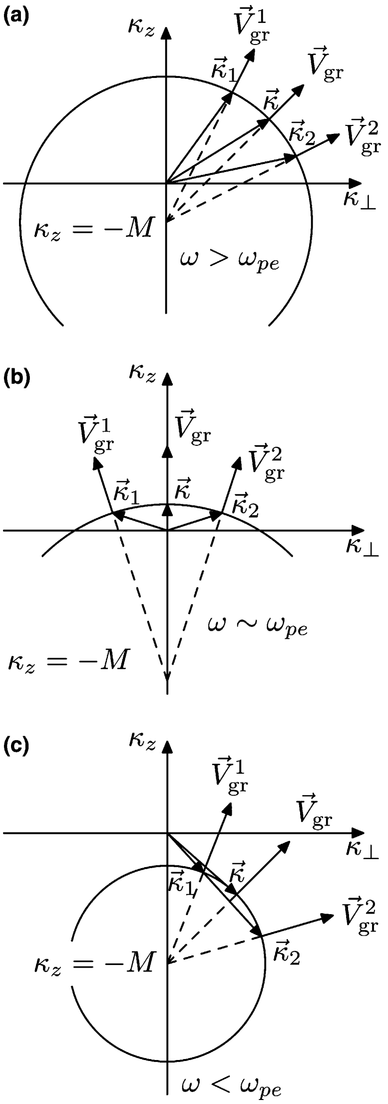

Fig. 2. (a) Wave number surfaces, $\omega > \omega_{\rm p}$. (b) Wave number surfaces, $\omega \sim \omega_{\rm p}$. (c) Wave number surfaces, $\omega < \omega_{\rm p}$.

The two Figs. 3(a) and (b) show the effective length computed according to (9) as a function of frequency above the plasma resonance at 31 kHz, with $M = 1/3$ and $v_{\rm T} = 1200$ km/s. Antenna position is fixed in space, with $\alpha = 30°$, $\phi_0 = 90°$ in Fig. 3(a) and $\alpha = 80°$, $\phi_0 = 15°$ in Fig. 3(b), just to show an example of changes of effective length with antenna orientation. The angle of

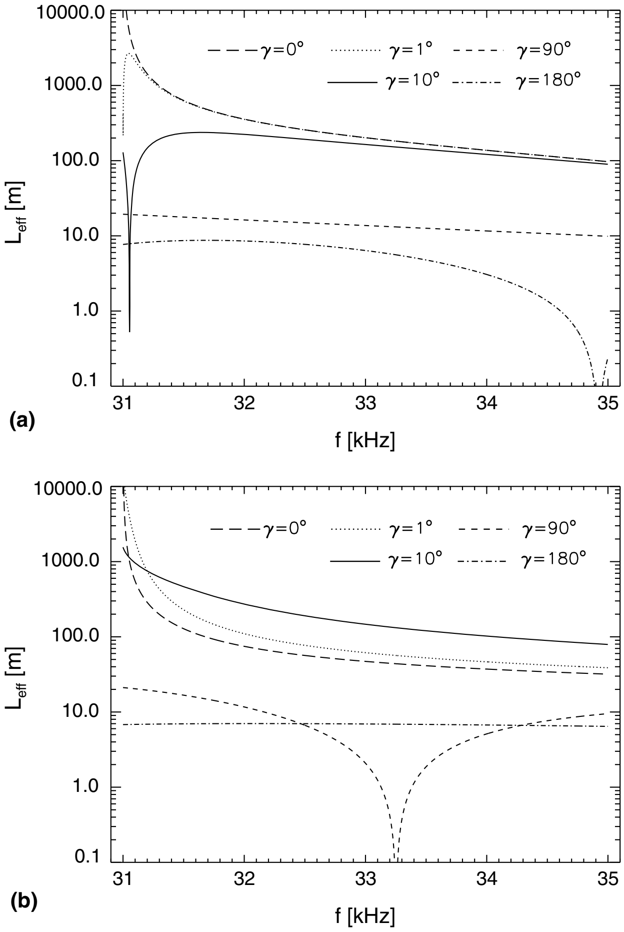

Fig. 3. (a) Antenna effective length vs. frequency, $f > f_p$, $\alpha = 30°$, $\phi_0 = 90°$. (b) Antenna effective length vs. frequency, $f > f_p$, $\alpha = 80°$, $\phi_0 = 15°$.

arrival γ_0 that the signal group velocity makes with the wind stream velocity is a parameter for the different curves, see the inset. The main result here is the strong growth of the effective length just above the plasma resonance for waves propagating under a small angle downstream, which results in corresponding strong enhancement of the voltage read by the receiver.

4. Conclusions

The theory presented relates quasi-thermal noise spectra to the antenna orientation with respect to the solar wind velocity. In principle this should allow for higher spatial/temporal resolution of plasma parameters deduced in course of interpretation of measured spectra. In practice we were obliged to choose one of the parameters in the fitting procedure as external, see above. This is due to the still unexplained discrepancy between measured and calculated spectra above the plasma resonance, which narrows the frequency band for a reliable fit, and to relatively low resolution in the spectra when compromising to obtain a reasonable angular resolution in antenna position on the spinning spacecraft. Nevertheless, our comparison of measured and computed QTN spectra shows that the spectral maximum does not correspond to the local plasma frequency accurately, it depends on the antenna position, and may be downshifted below the plasma frequency by about 1 kHz for typical parameters of the streaming solar wind. The cutoff and the height of the spectra make it possible to determine temperatures of the "cold" and "hot" plasma components. We performed a one case study to show the potential of our approach. Refinements in both the interpretation procedure and in the plasma model are being developed to be applied to a larger data set.

The growth of the antenna effective length by more than an order of magnitude close to the plasma resonance in a streaming plasma must be taken into account in interpretations of observations of quasi-harmonic waves coming from a distant source. In the experimental spectrograms (frequency vs. time, not shown here) these waves appear as narrow lines of high intensity. As it follows from our analysis this is a consequence of the sharp growth of the antenna effective length in case a coherent wave comes to the receiver downstream, in the direction of the solar wind velocity, with a frequency very close to the local plasma frequency. In particular, this means that the frequency of such a line in the spectrum of received radiation indicates the local electron plasma frequency with good accuracy, providing in this way additional means of diagnostics of solar wind parameters. The next study will include the frequencies below the plasma resonance to the cutoff and discussion about the value for this length at the resonance itself.

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References

Andronov, A.A., Chugunov, Yu.V. Quasisteady-state electric fields of sources in a dilute plasma. Sov. Physics Uspekhi (USA) 18 (5), 343–360, 1975.

Chugunov, Y.V. Receiving antenna in a magnetoplasma in the resonance frequency band. Radiophys. Quantum Electron. 44 (1–2), 151–160, 2001.

Chugunov, Y.V., Mareev, E.A., Fiala, V., James, H.G. Transmission of waves near the lower oblique resonance using dipoles in the ionosphere. Radio Sci. 38 (2), 1022, 2003.

Gurnett, D.A., Huff, R.L., Kirchner, D.L. The wide band plasma investigation. Space Sci. Rev. 79, 195–208, 1997.

Issautier, K., Meyer-Vernet, N., Moncuquet, M., Hoang, S., McComas, D.J. Quasi-thermal noise in a drifting plasma: theory and application to solar wind diagnostic on Ulysses. J. Geophys. Res. 104, 6691–6704, 1999.

James, H.G. Electrostatic resonance cone waves emitted by a dipole in the ionosphere. IEEE Trans. Antenn. Propag. 48, 1340–1348, 2000.

Meyer-Vernet, N., Hoang, S., Issautier, K., Maksimovic, M., Manning, R., Moncuquet, M., Stone, R.G. Measuring plasma parameters with thermal noise spectroscopy. In: Pfaff, R.F., Borovsky, J.E., Young, D.T. (Eds.), Measurement Techniques in Space Plasmas, Fields, Geophysical Monograph, 103. American Geophysical Union, USA, pp. 205–210, 2000.

Meyer-Vernet, N., Perche, C. Tool kit for antennae and thermal noise near the plasma frequency. J. Geophys. Res. 94, 2405–2415, 1989.